Indian Statistical Institute, Bangalore

M. Math.I Year, First Semester

Semestral Examination

Analysis of Several Variables (Back Paper)

Time: 3 hours Instructor: B.Bagchi

Maximum Marks 100

1. State and prove the inverse function theorem. [25]

2. The l^1- norm on $\mathbb{R}^d$ is defined by $\| x \|_1 = \sum_{i=1}^{d} |x_i|$.

   Compute the volume (Lebesgue measure) of the open unit ball with respect to this norm. [25]

3. Two norms $\| \cdot \|$ and $\| \widetilde{\|}$ on $\mathbb{R}^d$ are said to be equivalent if there are constants $c_1 > 0, c_2 > 0$ such that $c_1 \| x \| \leq \| x\widetilde{\|} \leq c_2 \| x \|$ for all $x \varepsilon \mathbb{R}^d$. Show that two norms on $\mathbb{R}^d$ are equivalent. [25]

4. Define Euler's Beta and Gamma functions. Show that they are related by the identify

$$\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}.$$

[25]